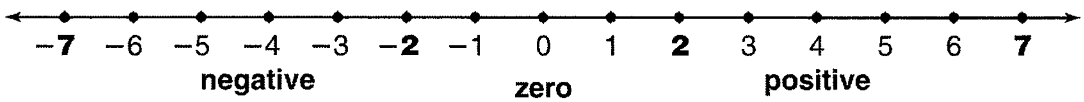
Name ____________________ Class ____________ Date ____________

Reteaching 1-1

Comparing and Ordering Integers

The numbers 2 and −2 are opposites. The numbers 7 and −7 are opposites.
Integers are the set of positive whole numbers, their opposites, and zero.



You can use the number line to compare integers.

−2 is less than 0.
$-2 < 0$

7 is greater than 2.
$7 > 2$

Numbers to the left are less. −2 is farther left than 0.	Numbers to the right are greater. 7 is farther to the right than 2.

Compare using <, >, or =.

1. 4 ☐ 2 **2.** −3 ☐ −2 **3.** 3 ☐ −4

4. −1 ☐ −2 **5.** 0 ☐ 5 **6.** 0 ☐ −4

7. −6 ☐ 4 **8.** −8 ☐ −2 **9.** 3 ☐ 0

10. −7 ☐ −10 **11.** −10 ☐ 10 **12.** 1 ☐ −1

Find the opposite of each number.

13. 8 ________ **14.** −5 ________ **15.** 147 ________

Find each sum.

16. 12 + 12 ________ **17.** −15 + 15 ________ **18.** 18 + 18 ________

19. 5 + 5 ________ **20.** −24 + 24 ________ **21.** −225 + 225 ________

Order the numbers from least to greatest.

22. −4, 5, −2, 0, 1

____

23. 6, −3, −5, 4, −6

____

24. 3, −5, 4, −4, −7, 0

____

25. 1, 3, −7, −6, 5, −2

____

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Reteaching 1-2

Adding and Subtracting Integers

Use these rules to add and subtract integers.

Adding Integers

Same Sign ← → Different Signs

Same Sign	Different Signs
• The sum of two positive integers is positive. Example: $6 + 16 = 22$ • The sum of two negative integers is negative. Example: $-9 + (-3) = -12$	• First find the absolute values of each number. • Then subtract the lesser absolute value from the greater. • The sum has the sign of the integer with the greater absolute value. Example: $-10 + 9 = -1$

Subtracting Integers

↓

- To subtract integers, add the opposite.
- Then following the rules for adding integers.
  Example: $6 - (-3) = 6 + 3 = 9$

Find each sum.

1. $8 + (-2)$ ______ **2.** $-9 + 4$ ______ **3.** $3 + (-2)$ ______

4. $-1 + 11$ ______ **5.** $12 + 13$ ______ **6.** $-9 + 5$ ______

7. $7 + 2$ ______ **8.** $-1 + (-7)$ ______ **9.** $-3 + 0$ ______

10. $-1 + (-1)$ ______ **11.** $6 + 5$ ______ **12.** $3 - (-2)$ ______

Complete.

13. $-3 - \ 4$ Change to addition: $-3 +$ ______ = ______

14. $5 - \ 2$ Change to addition: $5 +$ ______ = ______

15. $-6 - (-10)$ Change to addition: $-6 +$ ______ = ______

Find each difference.

16. $4 - 5$ ______ **17.** $-5 - 4$ ______ **18.** $-8 - (-7)$ ______

19. $19 - (-6)$ ______ **20.** $-10 - 12$ ______ **21.** $-12 - 10$ ______

22. $-4 - (-5)$ ______ **23.** $-2 - (-3)$ ______ **24.** $9 - (-7)$ ______

25. $0 - 3$ ______ **26.** $6 - 8$ ______ **27.** $0 - (-10)$ ______

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Reteaching 1-3

Multiplying and Dividing Integers

To multiply integers:

- If the signs are alike, the product is positive.

$$2 \cdot 3 = 6$$
$$-2 \cdot -3 = 6$$

- If the signs are different, the product is negative.

$$2 \cdot -3 = -6$$
$$-2 \cdot 3 = -6$$

To divide integers:

- If the signs are alike, the quotient is positive.

$$6 \div 3 = 2$$
$$-6 \div -3 = 2$$

- If the signs are different, the quotient is negative.

$$6 \div -3 = -2$$
$$-6 \div 3 = -2$$

Study these four examples. Write positive or negative to complete each statement.

$$\mathbf{7 \cdot 3 = 21} \qquad \mathbf{7 \cdot -3 = -21}$$
$$\mathbf{-7 \cdot -3 = 21} \qquad \mathbf{-7 \cdot 3 = -21}$$

1. When both integers are positive, the product is ____________.
2. When one integer is positive and one is negative, the product is ____________.
3. When both integers are negative, the product is ____________.

$$\mathbf{21 \div 3 = 7} \qquad \mathbf{-21 \div -3 = 7}$$
$$\mathbf{21 \div -3 = -7} \qquad \mathbf{-21 \div 3 = -7}$$

4. When both integers are positive, the quotient is ____________.
5. When both integers are negative, the quotient is ____________.
6. When one integer is positive and one is negative, the quotient is ____________.

Tell whether each product or quotient will be *positive* or *negative*.

7. $4 \cdot 7$ ____________
8. $-4 \cdot 7$ ____________
9. $-4 \cdot -7$ ____________
10. $4 \cdot -7$ ____________
11. $10 \cdot -4$ ____________
12. $-25 \div 5$ ____________
13. $-2 \cdot -2$ ____________
14. $100 \div 10$ ____________

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Reteaching 1-4

Fractions and Decimals

0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
	$\frac{1}{10}$	$\frac{1}{5}$			$\frac{1}{2}$	$\frac{3}{5}$				1		$1\frac{1}{5}$		$1\frac{2}{5}$

To change a fraction to a decimal, divide the numerator by the denominator.

$\frac{3}{5}$ Think: $3 \div 5$

$$\begin{array}{r} 0.6 \\ 5\overline{)3.0} \\ -30 \\ \hline 0 \end{array}$$

$\frac{3}{5} = 0.6$

To change a decimal to a fraction:

① Read the decimal to find the denominator. Write the decimal digits over 10, 100, or 1,000.

② 0.65 is 65 *hundredths* $\rightarrow \frac{65}{100}$

Use the GCF to write the fraction in simplest form.

The GCF of 65 and 100 is 5.

$$\frac{65}{100} = \frac{65 \div 5}{100 \div 5} = \frac{13}{20}$$

Write each fraction as a decimal.

1. $\frac{4}{5}$ = ______
2. $\frac{3}{4}$ = ______
3. $\frac{1}{6}$ = ______
4. $\frac{1}{4}$ = ______
5. $\frac{2}{3}$ = ______
6. $\frac{7}{10}$ = ______
7. $\frac{5}{9}$ = ______
8. $\frac{1}{5}$ = ______
9. $\frac{3}{8}$ = ______

Write each decimal as a mixed number or fraction in simplest form.

10. 0.4 = ______
11. 0.75 = ______
12. 1.5 = ______
13. 0.35 = ______
14. 2.7 = ______
15. 1.8 = ______
16. 0.625 = ______
17. 0.78 = ______
18. 0.88 = ______

Order from least to greatest.

19. $2.\overline{6}, \frac{13}{6}, 2\frac{5}{6}$ ____________________
20. $2.\overline{02}, 2\frac{1}{200}, 2.0202$ ____________________
21. $\frac{5}{4}, 1\frac{4}{5}, 1.\overline{4}$ ____________________

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Reteaching 1-5

Rational Numbers

A **rational number** is a number that can be written as a quotient of two integers, where the divisor is not zero.

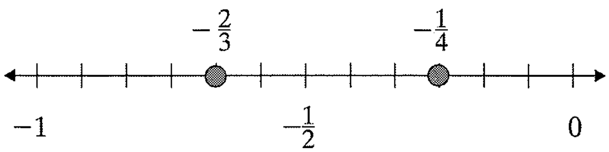
A negative rational number can be written in three different ways.

$$-\frac{2}{3} = -\frac{2}{3} = -\frac{2}{3}$$

Comparing Negative Rational Numbers

Compare $-\frac{2}{3}$ and $-\frac{1}{4}$.

Method 1 Use a number line. Graph both points on a number line and see which is farther to the left.



Since $-\frac{2}{3}$ is farther to the left, $-\frac{2}{3} < -\frac{1}{4}$.

Method 2 Use the lowest common denominator.

$$-\frac{2}{3} = \frac{-2}{3} = \frac{-2 \times 4}{3 \times 4} = \frac{-8}{12} \qquad -\frac{1}{4} = \frac{-1}{4} = \frac{-1 \times 3}{4 \times 3} = \frac{-3}{12}$$

Since $\frac{-8}{12} < \frac{-3}{12}$, then $-\frac{2}{3} < -\frac{1}{4}$.

Compare. Use <, >, or =.

1. $-\frac{4}{9}$ ☐ $-\frac{2}{3}$
2. -1 ☐ $-\frac{4}{5}$
3. $-\frac{7}{8}$ ☐ $-\frac{1}{8}$
4. $-\frac{1}{3}$ ☐ $-\frac{5}{6}$
5. $-\frac{2}{5}$ ☐ $-\frac{1}{10}$
6. $-\frac{2}{8}$ ☐ $-\frac{1}{4}$

Order from least to greatest.

7. $-\frac{1}{3}$, 0.3, −0.35, $-\frac{3}{10}$

   ____

8. $\frac{1}{5}$, −0.25, 0.21, $\frac{3}{10}$

   ____

9. You and your brother invested an equal amount of money in a college savings plan. In the last quarter your investment was worth $1\frac{5}{6}$ of its original value. Your brother's investment was worth 1.85 of its original value. Whose investment is worth more?

   ____

Name ______________________ Class ______________ Date ______________

Reteaching 1-6

Adding and Subtracting Rational Numbers

Use these rules to add and subtract rational numbers.

Adding and Subtracting Integers

Same Sign	Different Signs
• The sum of two positive rational numbers is positive. Example: $15.6 + 4.5 = 20.1$ Example: $\frac{2}{9} + \frac{8}{9} = 1\frac{1}{9}$ • The sum of two negative rational numbers is negative. Example: $-3.42 + (-5.74) = -9.16$ Example: $-1\frac{3}{4} + \left(-4\frac{3}{4}\right) = -6\frac{1}{2}$	• First find the absolute values of each addend. • Then subtract the lesser absolute value from the greater. • The sum has the sign of the addend with the greater absolute value. Example: $-25.8 + 17.3 = -8.5$ Example: $-2\frac{1}{2} + 1\frac{1}{4} = -1\frac{1}{4}$

Subtracting Rational Numbers

- To subtract rational numbers, add the opposite.
- Then following the rules for adding rational numbers.
  Example: $-9.25 - (-3.4) = -9.25 + 3.4 = -5.85$
  Example: $4 - \left(-2\frac{3}{10}\right) = 4 + \left(2\frac{3}{10}\right) = 6\frac{3}{10}$

Find each sum.

1. $43.2 + 26.7$ ______
2. $-81.22 + 14.9$ ______
3. $-4.8 + (-53.5)$ ______
4. $2\frac{5}{9} + 3\frac{4}{9}$ ______
5. $-2\frac{3}{5} + 1\frac{1}{5}$ ______
6. $-6\frac{1}{3} + \left(-7\frac{1}{3}\right)$ ______

Find each difference.

7. $15.64 - 8.5$ ______
8. $-0.392 - 0.26$ ______
9. $-5.4 - (-1.6)$ ______
10. $6 - 5\frac{5}{6}$ ______
11. $-4\frac{3}{4} - 2\frac{1}{4}$ ______
12. $-7\frac{4}{5} - \left(-3\frac{3}{5}\right)$ ______

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Reteaching 1-7

Multiplying Rational Numbers

Remember these rules when multiplying rational numbers.

1. When both factors are positive, the product is positive.
   Multiply: $\left(2\frac{2}{3}\right)\left(1\frac{5}{8}\right) = \left(\frac{8}{3}\right)\left(\frac{13}{8}\right) = \frac{104}{24} = 4\frac{1}{3}$

2. When both factors are negative, the product is positive.

   Multiply: $(-4.35)(-2.44) = 10.614$

3. When both factors have different signs, the product is negative.
   Multiply: $-\frac{3}{4} \times \frac{2}{5} = \frac{-3 \times 2}{4 \times 5} = \frac{-6}{20} = -\frac{6}{20} = -\frac{3}{10}$

Find each product. Write the product in simplest form.

1. 2.8×0.05 ____________
2. $\frac{5}{8} \cdot \frac{2}{5}$ ____________
3. $1.45 \cdot 0.7$ ____________
4. $2\frac{3}{5} \cdot \frac{7}{8}$ ____________
5. $(-2.07)(-4.9)$ ____________
6. $\frac{5}{12} \cdot \left(-\frac{3}{10}\right)$ ____________
7. $9.3(-0.56)$ ____________
8. $\frac{1}{2} \times 5\frac{1}{6}$ ____________
9. $0.006(3.75)$ ____________
10. $-1\frac{2}{3} \times 5$ ____________
11. -3.8×912 ____________
12. $\left(-2\frac{3}{5}\right)\left(-\frac{1}{4}\right)$ ____________

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Reteaching 1-8

Dividing Rational Numbers

Divide: 38.25 ÷ 1.5.		Divide: $3\frac{3}{4} \div 1\frac{2}{5}$.	
1. Rewrite the problem with a whole number divisor.	$1.5\overline{)38.25}$ ↓	**1.** Rewrite mixed numbers as improper fractions as needed.	$\frac{15}{4} \div \frac{7}{5}$
2. Place the decimal point in the quotient.	$1.5.\overline{)38.2.5}$ ↑ ↑ **Move 1 place each.**	**2.** Multiply by the reciprocal of the divisor.	$\frac{15}{4} \cdot \frac{5}{7}$
3. Divide. Then check.	$15\overline{)382.5}$ = 25.5; −30, 82, −75, 75, −75, 0. $25.5 \times 15 = 382.5$ ✔ **Multiply to check.**	**3.** Multiply numerators. Multiply denominators.	$\frac{15 \cdot 5}{4 \cdot 7} = \frac{75}{28}$
		4. Simplify.	$\frac{75}{28} = 2\frac{19}{28}$

Find each quotient. Simplify your answers.

1. $1\frac{5}{8} \div \frac{5}{8}$ ____________

2. $-43.55 \div 6.5$ ____________

3. $-\frac{2}{5} \div \frac{4}{25}$ ____________

4. $-0.072 \div 0.8$ ____________

5. $-12\frac{4}{5} \div \left(-1\frac{1}{15}\right)$ ____________

6. $340.2 \div -4.2$ ____________

7. $-15 \div \frac{1}{2}$ ____________

8. $12.6 \div 0.21$ ____________

9. $-7\frac{1}{3} \div 2\frac{1}{5}$ ____________

10. $-11.1 \div (-37)$ ____________

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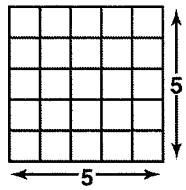
Reteaching 1-9

Irrational Numbers and Square Roots

- The *square* of 5 is 25.
  $5 \cdot 5 = 5^2 = 25$
- The *square root* of 25 is 5 because $5^2 = 25$.

$\sqrt{25} = 5$



$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
} *perfect squares*

Example: You can use a calculator to find square roots.
Find $\sqrt{36}$ and $\sqrt{21}$ to the nearest tenth.

36 [√] = 6   21 [√] ≈ 4.5825757 ≈ 4.6

You can estimate square roots like $\sqrt{52}$ and $\sqrt{61}$.

Perfect squares	49		$\sqrt{49} = 7$		$\sqrt{49} = 7$
	52	Estimate	$\sqrt{52} \approx 7$	Estimate	$\sqrt{61} \approx 8$
	64		$\sqrt{64} = 8$		$\sqrt{64} = 8$

Find each square root. Estimate to the nearest integer if necessary.
Use ≈ to show that a value is estimated.

1. $\sqrt{16}$ ________ **2.** $\sqrt{85}$ ________ **3.** $\sqrt{26}$ ________ **4.** $\sqrt{36}$ ________

5. $\sqrt{98}$ ________ **6.** $\sqrt{40}$ ________ **7.** $\sqrt{100}$ ________ **8.** $\sqrt{18}$ ________

9. $\sqrt{5}$ ________ **10.** $\sqrt{121}$ ________ **11.** $\sqrt{68}$ ________ **12.** $\sqrt{144}$ ________

13. $\sqrt{29}$ ________ **14.** $\sqrt{64}$ ________ **15.** $\sqrt{37}$ ________ **16.** $\sqrt{75}$ ________

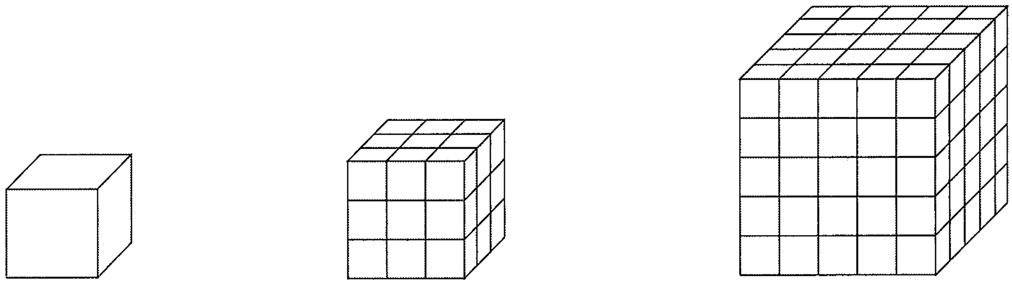
17. If a whole number is not a perfect square, its square root is an *irrational number.* List the numbers from exercises 1–16 that are irrational.

____

____

Reteaching 1-10

Cube Roots



The cube of 1 is 1.	The cube of 3 is 27.	The cube of 5 is 125.
$1 \times 1 \times 1 = 1^3 = 1$	$3 \times 3 \times 3 = 3^3 = 27$	$5 \times 5 \times 5 = 5^3 = 125$

perfect cubes

$1^3 = 1$ $3^3 = 27$ $5^3 = 125$

Example: You can solve cube root equations: $x^3 = \frac{27}{216}$

$\sqrt[3]{x^3} = \sqrt[3]{\frac{27}{216}}$ ← **Find the cube root of each side.**

$= \frac{\sqrt[3]{27}}{\sqrt[3]{216}}$ ← **Find the cube root of the numerator and denominator.**

$x = \frac{3}{6} = \frac{1}{2}$ ← **Simplify.**

Find the cube root of each number.

1. 729 ________

2. 125 ________

3. 512 ________

4. -64 ________

5. $\frac{1}{216}$ ________

6. $\frac{125}{1000}$ ________

Solve each equation by finding the value of *x*.

7. $x^3 = 27$ ________

8. $x^3 = 1{,}728$ ________

9. $x^3 = \frac{343}{729}$ ________

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Reteaching 2-1

Scientific Notation

To write a number such as 67,000 in *scientific notation,* move the decimal point to form a number between 1 and 10. The number of places moved shows which power of 10 to use.

- Write 67,000 in scientific notation.

  6.7 is between 1 and 10. So, move the decimal point in 67,000 to the left 4 places and multiply by 10^4.

  $67{,}000 = 6.7 \times 10^4$

To write scientific notation in *standard form,* look at the exponent. The exponent shows the number of places and the direction to move the decimal point.

- Write 8.5×10^5 in standard form.

  The exponent is positive 5, so move the decimal point 5 places to the right.

  $8.5 \times 10^5 = 850{,}000$

Write each number in scientific notation.

1. 6,500 ______
2. 65,000 ______
3. 6,520 ______
4. 345 ______
5. 29,100 ______
6. 93,000,000 ______
7. 200 ______
8. 2,300 ______
9. 23,000 ______

Write each number in standard form.

10. 4×10^4 ______
11. 4×10^5 ______
12. 3.6×10^3 ______
13. 4.85×10^4 ______
14. 4.05×10^2 ______
15. 7.1×10^5 ______
16. 4×10^2 ______
17. 1.3×10^2 ______
18. 7×10^1 ______
19. 1.81×10^3 ______
20. Jupiter orbits at an average of 7.783×10^8 kilometers from the Sun. ______

Which number is greater?

21. 5×10^2 or 2×10^5 ______
22. 2.1×10^3 or 2.1×10^6 ______
23. 6×10^{10} or 3×10^9 ______
24. 3.6×10^1 or 3.6×10^3 ______

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Reteaching 2-2

Exponents and Multiplication

- To multiply numbers or variables with the same base, add the exponents.

Simplify $3^2 \cdot 3^4$	Simplify $n^3 \cdot n^4$	Simplify $(-4)^3 \cdot (-4)^5$
$3^2 \cdot 3^4 = 3^{(2+4)}$	$n^3 \cdot n^4 = n^{(3+4)}$	$(-4)^3 \cdot (-4)^5 = (-4)^{(3+5)}$
$= 3^6$	$= n^7$	$= (-4)^8$

- You can also simplify expressions with exponents.

$6x^2 \cdot -2x^5 = 6 \cdot -2 \cdot x^2 \cdot x^5$ ← Use the Commutative Property of Multiplication

$= -12x^{(2+5)}$ ← Add the exponents.

$= -12x^7$ ← Simplify.

Write each expression using a single exponent.

1. $5^3 \cdot 5^4$ ______
2. $a^2 \cdot a^5$ ______
3. $(-8)^4 \cdot (-8)^5$ ______
4. $n^6 \cdot n^2$ ______
5. $m^3 \cdot m^6$ ______
6. $(-7)^4 \cdot (-7)^2$ ______
7. $(-3)^2 \cdot (-3)^2$ ______
8. $2^5 \cdot 2^2$ ______
9. $c^5 \cdot c^3$ ______

Find each product. Write the answer in scientific notation.

10. $2x^3 \cdot x^2$ ______
11. $-4x^3 \cdot 2x^4$ ______
12. $3a^3 \cdot a$ ______
13. $-x^2 \cdot 2x^3$ ______
14. $-5m^2 \cdot -2m^4$ ______
15. $x^8 \cdot x^4$ ______

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Reteaching 2-3

Multiplying with Scientific Notation

- To multiply numbers in scientific notation.

Find the product $(5 \times 10^4)(7 \times 10^5)$. Write the result in scientific notation.

$(5 \times 10^4)(7 \times 10^5)$		
$(5 \cdot 7)(10^4 \cdot 10^5)$	←	Use the Associative and Commutative properties.
$35 \times (10^4 \cdot 10^5)$	←	Multiply 5 and 7.
$35 \times 10^{4+5}$	←	Add the exponents for the powers of 10.
35×10^9		
$3.5 \times 10^1 \times 10^9$	←	Write 35 in scientific notation.
3.5×10^{10}	←	Add the exponents.

Find each product. Write the answer in scientific notation.

1. $(3 \times 10^4)(5 \times 10^3)$ ____________
2. $(2 \times 10^3)(7 \times 10^6)$ ____________
3. $(8 \times 10^2)(5 \times 10^2)$ ____________
4. $(9 \times 10^4)(7 \times 10^4)$ ____________
5. $(4 \times 10^2)(7 \times 10^5)$ ____________
6. $(8 \times 10^3)(4 \times 10^5)$ ____________

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Reteaching 2-4

Exponents and Division

To divide powers with the same base, subtract exponents.

$\frac{8^6}{8^4} = 8^{6-4}$ $\quad$ $\frac{a^5}{a^3} = a^{5-3}$

$= 8^2$ $\quad$ $= a^2$

$= 64$

- For any nonzero number a, $a^0 = 1$.

$3^0 = 1$ $\quad$ $(-6)^0 = 1$ $\quad$ $4t^0 = 4(1) = 4$

- For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$.

$2^{-4} = \frac{1}{2^4}$ $\quad$ $3c^{-2} = \frac{3}{c^2}$ $\quad$ $\frac{5^3}{5^6} = 5^{3-6}$ $\quad$ $\frac{10z^3}{5z} = 2z^{3-1}$

$= \frac{1}{16}$ $\quad$ $= 5^{-3}$ $\quad$ $= 2z^2$

$= \frac{1}{5^3}$

$= \frac{1}{125}$

Simplify each expression.

1. $\frac{6^5}{6^3} =$ ______________ **2.** $(-4)^5 \div (-4)^3 =$ ______ **3.** $(-3)^{-2} =$ ______________

4. $\frac{2^5}{2^7} =$ ______________ **5.** $(-8)^0 =$ ______________ **6.** $\frac{5^0}{5^2} =$ ______________

7. $\frac{(-6)^4}{(-6)^6} =$ ______________ **8.** $7^3 \div 7^5 =$ ______________ **9.** $9^8 \div 9^{10} =$ ______________

Simplify each expression. Write your answer using only positive exponents.

10. $w^8 \div w^3 =$ ______________ **11.** $x^6 \div x^1 =$ ______________ **12.** $\frac{d^7}{d^3} =$ ______________

13. $\frac{w^2}{w^6} =$ ______________ **14.** $4c^5 \div c^8 =$ ______________ **15.** $\frac{8x^2}{4x^5} =$ ______________

16. $8a^4 \div 2a^2 =$ ______________ **17.** $6w^2 \div 2w^5 =$ ______________ **18.** $\frac{6x^6}{3x^9} =$ ______________

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Reteaching 2-5

Dividing with Scientific Notation

You can separate the coefficients and powers of ten to divide numbers in scientific notation.

$(8.4 \times 10^6) \div (2.5 \times 10^4) = \frac{8.4 \times 10^6}{2.5 \times 10^4}$ ← Write a fraction.

$= \frac{8.4}{2.5} \times \frac{10^6}{10^4}$ ← Separate the coefficients and the power of ten.

$\approx 3.4 \times \frac{10^6}{10^4}$ ← Divide the coefficients.

$\approx 3.4 \times 10^2$ ← Subtract the exponents.

You can divide numbers in standard form by numbers in scientific notation.

$(9.2 \times 10^4) \div 4.8 = \frac{9.2 \times 10^4}{4.8}$ ← Write a fraction.

$= \frac{9.2}{4.8} \times 10^4$ ← Write as a product of quotients and a power of ten.

$\approx 1.9 \times 10^4$ ← Divide.

You can divide numbers in scientific notation by numbers in standard form.

$6.8 \div (3.9 \times 10^2) = \frac{6.8}{3.9 \times 10^2}$ ← Write a fraction.

$\approx \frac{6.8}{3.9} \times \frac{1}{10^2}$ ← Write as a product of quotients and a power of ten.

$\approx 1.7 \times 10^{-2}$

Divide. Write each quotient in scientific notation. Round answers to the nearest tenth.

1. $\frac{6.4 \times 10^5}{1.8 \times 10^3}$ ____________________

2. $\frac{7.4 \times 10^4}{3.3}$ ____________________

3. $\frac{8}{2.6 \times 10^2}$ ____________________

4. $\frac{9.2 \times 10^4}{5.9 \times 10^2}$ ____________________

5. $\frac{6.5 \times 10^8}{8.9}$ ____________________

6. $\frac{12.2}{6.3 \times 10^4}$ ____________________

7. $\frac{5.4 \times 10^2}{0.5}$ ____________________

8. $\frac{3.4 \times 10^8}{1.2 \times 10^6}$ ____________________

Name ______________________ Class ______________ Date ______________

Reteaching 3-1

Evaluating and Writing Algebraic Expressions

To evaluate an *expression*, substitute a value for the *variable* and compute.

Evaluate $5y - 8$ for $y = 7$.

$5y - 8$

$5 \times 7 - 8$ ← **Substitute *y* with 7.**

$35 - 8 = 27$ ← **Compute.**

You can use key words to write a word phrase for an algebraic expression.

$a + 5$ → *a* plus 5
or *a* increased by 5

$2n$ → the product of 2 and *n*
or 2 times *n*

Evaluate each expression using the values $m = 3$ and $x = 8$.

1. $4m + 9$
Substitute *m*: 4 × ______ + 9
Compute: ______ + 9 = ______

2. $4x - 7$
Substitute *x*: 4 × ______ − 7
Compute: ______ − 7 = ______

3. $5x + x$
Substitute *x*: 5 × ______ + ______
Compute: ______ + ______ = ______

4. $x + 2m$
Substitute *x* and *m*: ______ + 2 × ______
Compute: ______ + ______ = ______

Evaluate each expression using the values $y = 4$, $z = 8$, and $p = 10$.

5. $3y + 6 =$ ______

6. $4z - 2 =$ ______

7. $p + 2p =$ ______

8. $3z \times z =$ ______

Write a word phrase for each algebraic expression.

9. $9 + x$
____

10. $6x$
____

11. $x - 8$
____

12. $\frac{x}{5}$
____

Write an algebraic expression for each word phrase.

13. *x* newspapers plus 10
____

14. 4 less than *x* teabags
____

15. 3 more than *x* envelopes
____

16. 6 times *x* school buses
____

Name ______________________ Class ______________ Date ______________

Reteaching 3-2

Simplifying Expressions

A *term* is a number, a variable, or the product of a number and variable(s).The two terms in $-2x + 4y$ are $-2x$ and $4y$.

A *coefficient* is a numerical factor of a term with a variable. In $5x$ and $(3 + 1)y$, the coefficients are 5 and $(3 + 1)$.

Terms with exactly the same variable factor are called *like terms*. In $-2x + 6y + 3x$, $-2x$ and $3x$ are like terms.

One way to *combine like terms* is by addition or subtraction.

- Add to combine like terms in $5a + a$.
  $5a + a = (5 + 1)a = 6a$
- Subtract to combine like terms in $7b - 10b$.
  $7b - 10b = (7 - 10)b = -3b$

To *simplify* an expression, combine its like terms.

Perform as many of its operations as possible.

Simplify: $6m + 10 - 2m + 4$

$= (6m - 2m) + (10 + 4)$

$= 4m + 14$

Simplify: $3(c - 6)$

$= 3c - 3(6)$

$= 3c - 18$

You can use the Distributive Property to rewrite an addition expression as a product of two factors. This process is called factoring. Use the greatest common factor (GCF) so the expression is factored completely.

Factor $9x + 12$.

GCF of 9 and 12 is 3. ← **Identify the GCF.**

$9x + 12 = 3 \cdot 3x + 3 \cdot 4$ ← **Factor each term by the GCF.**

$= 3(3x + 4)$ ← **Distributive Property**

The factored expression is $3(3x + 4)$.

Factor $15y - 5$.

GCF of 15 and 5 is 5. ← **Identify the GCF.**

$15y - 5 = 5 \cdot 3y - 5 \cdot 1$ ← **Factor each term by the GCF.**

$= 5(3y - 1)$ ← **Distributive Property**

The factored expression is $5(3y - 1)$.

Simplify each expression.

1. $7(6 + p) =$ ______________
2. $6n + 2(4n + 5) =$ ______________
3. $3(0.3x + 0.1) + 0.2x =$ ______________
4. $-6x - 8 + 3x - 14 =$ ______________
5. $2x + 9 - 0.74x + 2.24 =$ ______________
6. $-18(b - 1) - 18 =$ ______________
7. $-0.4p - 4.2 + 6.2p - 0.9 =$ ______________
8. $\frac{1}{5}(h - 15) + 12h - 8 =$ ______________
9. $-\frac{3}{4}(d + 6) - 2d + 7 =$ ______________
10. $-5.6(q - 3.2) - 4.5q + 3.6 - q =$ ______________

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Reteaching 3-3

Solving One-Step Equations

Follow these steps to solve equations by adding and subtracting.

	Solve: $n + (-2) = 11$	Solve: $n - 6 = -36$
① Use the inverse operation on both sides of the equation.	$n + (-2) - (-2) = 11 - (-2)$	$n - 6 + 6 = -36 + 6$
② Simplify.	$n = 13$	$n = -30$
③ Check.	$n + (-2) = 11$ $13 + (-2) \stackrel{?}{=} 11$ $11 = 11$ ✔	$n - 6 = -36$ $-30 - 6 \stackrel{?}{=} -36$ $-36 = -36$ ✔

Follow these steps to solve equations by multiplying and dividing.

	Solve: $\frac{t}{5} = -7$	Solve: $-2x = 8$
① Use the inverse operation on both sides of the equation.	$(5)\frac{t}{5} = (5)(-7)$	$\frac{-2x}{-2} = \frac{8}{-2}$
② Simplify.	$t = -35$	$x = -4$
③ Check.	$\frac{t}{5} = -7$ $\frac{-35}{5} \stackrel{?}{=} -7$ $-7 = -7$ ✔	$-2x = 8$ $-2(-4) \stackrel{?}{=} 8$ $8 = 8$ ✔

Solve each equation. Check each answer.

1. $n + 6 = 8$

$n + 6 - 6 = 8 -$ ______

$n =$ ______

2. $n - 3 = 20$

$n - 3 +$ ______ $= 20 + 3$

$n =$ ______

3. $\frac{a}{2} = -16$

$(\square)\frac{a}{2} = (\square)(-16)$

$a =$ ______

4. $-2w = -4$

$\frac{-2w}{\square} = \frac{-4}{\square}$

$w =$ ______

Use a calculator, pencil and paper, or mental math. Solve each equation.

5. $n + 1 = 17$ ______

6. $n - (-6) = 7$ ______

7. $n - 8 = -12$ ______

8. $\frac{x}{4} = -1$ ______

9. $-5w = 125$ ______

10. $\frac{m}{-8} = 10$ ______

Name ______________________ Class ______________ Date ______________

Reteaching 3-4

Exploring Two-Step Equations

You can change a word expression into an algebraic expression by converting the words to variables, numbers, and operation symbols.

To write a two-step algebraic expression for *seven more than three times a number,* follow these steps.

①	Define the variable.	Let n represent the number.
②	Ask yourself if there are any key words.	"More than" means add and "times" means multiply.
③	Write an algebraic expression.	$7 + 3 \cdot n$
④	Simplify.	$7 + 3n$

Define a variable and write an algebraic expression for each phrase.

1. 3 inches more than 4 times your height ______________
2. 4 less than 6 times the weight of a turkey ______________
3. 8 more than one-half the number of miles run last week ______________

Solve.

4. Three friends pay $4 per hour to rent a paddleboat plus $5 for snacks. Write an expression for the total cost of rental and snacks. Then evaluate the expression for 2 hours.

____

5. A lawn care service charges $10 plus $15 per hour to mow and fertilize lawns. Write an expression for the total cost of having your lawn mowed and fertilized. Then evaluate the expression for 4 hours.

____

Solve each equation using number sense.

6. $4x - 10 = 30$ ______________

7. $2n - 7 = 13$ ______________

8. $\frac{s}{3} + 2 = 4$ ______________

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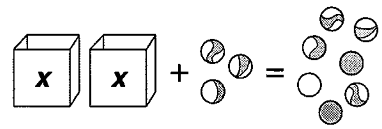
Reteaching 3-5

Solving Two-Step Equations

The marbles and boxes represent this equation.

$$2x + 3 = 7$$



The variable x stands for the number of marbles (unseen) in each box.

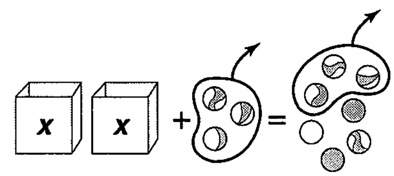
To solve the equation, follow these steps.

There are the same number of marbles on each side and the same number of marbles in each box.

Step 1

Subtract the extra marbles from both sides.

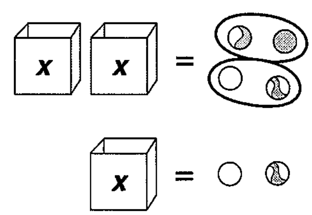
$$2x + 3 - 3 = 7 - 3$$
$$2x = 4$$



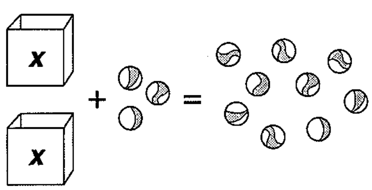
Step 2

Divide the number of marbles by 2, the number of boxes.

$$\frac{2x}{2} = \frac{4}{2}$$
$$x = 2$$

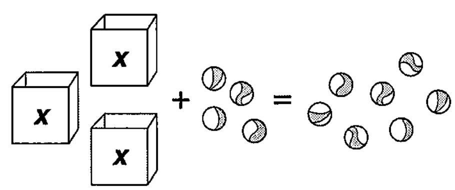


Write and solve an equation for each situation.

1. 

_______ + _______ = _______

x = _______

2. 

_______ + _______ = _______

x = _______

Complete to solve each equation.

3. $5x + 7 = 2$

$5x + 7 -$ _______ $= 2 -$ _______

$\frac{5x}{\square} = \frac{-5}{\square}$

x = _______

4. $2x - 1 = 9$

$2x - 1 +$ _______ $= 9 +$ _______

$\frac{2x}{\square} = \frac{10}{\square}$

x = _______

Solve each equation.

5. $4x + 7 = 15$ _______

6. $3b - 5 = 13$ _______

7. $5t - 2 = -17$ _______

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Reteaching 3-6

Solving Multi-Step Equations

Combining terms can help solve equations.

Solve: $5n + 6 + 3n = 22$

$5n + 3n + 6 = 22$ ← Commutative Property

$8n + 6 = 22$

$8n + 6 - 6 = 22 - 6$

$8n = 16$

$\frac{8n}{8} = \frac{16}{8}$

$n = 2$

Check: $5n + 6 + 3n = 22$

$5(2) + 6 + 3(2) \stackrel{?}{=} 22$

$22 = 22$ ✔

Sometimes you need to distribute a term in order to simplify.

Solve: $4(x + 2) = 28$

$4x + 8 = 28$ ← Distributive Property

$4x = 20$

$\frac{4x}{4} = \frac{20}{4}$

$x = 5$

Check: $4(n + 2) = 28$

$4(5 + 2) \stackrel{?}{=} 28$

$28 = 28$ ✔

Solve each equation. Check the solution.

1. $a - 4a = 36$

   $a =$ ______________

2. $3b - 5 - 2b = 5$

   $b =$ ______________

3. $5n + 4 - 8n = -5$

   $n =$ ______________

4. $12k + 6 = 10$

   $k =$ ______________

5. $3(x - 4) = 15$

   $x =$ ______________

6. $y - 8 + 2y = 10$

   $y =$ ______________

7. $3(s - 10) = 36$

   $s =$ ______________

8. $-15 = p + 4p$

   $p =$ ______________

9. $2g + 3g + 5 = 0$

   $g =$ ______________

10. $6c + 4 - c = 24$

    $c =$ ______________

11. $3(x - 2) = 15$

    $x =$ ______________

12. $4y + 9 - 7y = -6$

    $y =$ ______________

13. $4(z - 2) + z = -13$

    $z =$ ______________

14. $24 = -2(b - 3) + 8$

    $b =$ ______________

15. $17 = 3(g + 3) - g$

    $g =$ ______________

Name ____________________ Class ____________ Date ____________

Reteaching 3-7

Solving Equations With Variables on Both Sides

When an equation has a variable on both sides, add or subtract to get the variable on one side.

Solve: $-6m + 45 = 3m$

$-6m + 6m + 45 = 3m + 6m$ ← Add 6m to each side.

$45 = 9m$

$\frac{45}{9} = \frac{9m}{9}$

$5 = m$

Check: $-6m + 45 = 3m$

$-6(5) + 45 \stackrel{?}{=} 3(5)$

$15 = 15$ ✔

Sometimes you need to distribute a term in order to simplify.

Solve: $5(x - 3) = 32 - 2$

$5x - 15 = 32 - 2$ ← Distributive Property

$5x - 15 = 30$

$5x = 45$

$\frac{5x}{5} = \frac{45}{5}$

$x = 9$

Check: $5(x - 3) = 32 - 2$

$5(9 - 3) = 32 - 2$

$30 = 30$ ✔

Solve each equation. Check the solution.

1. $9j + 35 = 4j$

   $j =$ ____________

2. $13s = 2s - 66$

   $s =$ ____________

3. $2(5t - 4) = 12t$

   $t =$ ____________

4. $6q = 6(4q + 1)$

   $q =$ ____________

5. $7(t - 2) - t = 4$

   $t =$ ____________

6. $6w + 4 = 4w + 1$

   $w =$ ____________

7. $2(2q + 1) = 3(q - 2)$

   $q =$ ____________

8. $5z - 3 = 2(z - 3)$

   $z =$ ____________

9. $4(x + 0) = 2x + 6$

   $x =$ ____________

10. $5(k - 4) = 4 - 3k$

    $k =$ ____________

11. $8 - m - 3m = 16$

    $m =$ ____________

12. $6n + n + 14 = 0$

    $n =$ ____________

13. $7(p + 1) = 9 - p$

    $p =$ ____________

14. $41 - q = 3(q - 5)$

    $q =$ ____________

15. $25 + 2t = 5(t + 2)$

    $t =$ ____________

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Reteaching 3-8

Types of Solutions of Linear Equations

If an equation is true for all values of *x*:

$$a = a$$

infinitely many solutions

$4x + 8 = 4(x + 2)$	
$4x + 8 = 4x + 8$	Distributive Property
$4x + 8 - 4x = 4x + 8 - 4x$	Subtract
$8 = 8$	Simplify

If an equation is true for one value of *x*:

$$x = a$$

one solution

$5x - 3 = -3x + 5$	
$5x - 3 + 3 = -3x + 5 + 3$	Add
$5x = -3x + 8$	Simplify
$5x + 3x = -3x + 3x + 8$	Add
$8x = 8$	Divide
$x = 1$	

If an equation is not true for any values of *x*:

$$a = b$$

no solutions

$6x + 2 = 6(x - 1)$	
$6x + 2 = 6x - 6$	Distributive Property
$6x - 6x + 2 = 6x - 6x - 6$	Subtract
$2 = -6$	

Tell whether each equation has one solution, infinitely many solutions, or no solution.

1. $3x - 2 = x + 6$

____

2. $5x - 10 = 5(x - 2)$

____

3. $6x - 1 = 6(x + 2)$

____

4. $8(x + 2) = 8x + 16$

____

5. $2(x - 3) = 2x + 4$

____

6. $x + 4 = 3(x - 2)$

____

7. $x + 5 = 2x + 2$

____

8. $9(x + 1) = 9x + 9$

____

9. $6x + 8 = x - 2$

____

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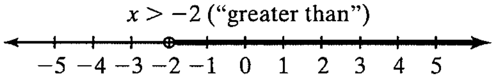
Reteaching 4-1

Graphing and Writing Inequalities

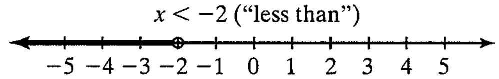
Two expressions separated by an inequality sign form an **inequality.** An inequality shows that the two expressions *are not* equal. Unlike the equations you have worked with, an inequality has many solutions.

The **solutions of an inequality** are the values that make the inequality true. They can be graphed on a number line. Use a closed circle (•) for ≤ and ≥ and an open circle (**o**) for > and <. For example:

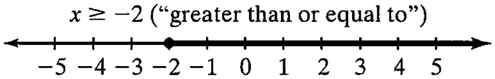
$x > -2$ ("greater than")



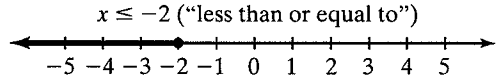
$x < -2$ ("less than")



$x \geq -2$ ("greater than or equal to")

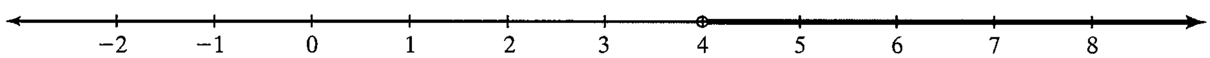


$x \leq -2$ ("less than or equal to")



Graph the inequality $x > 4$.

The inequality is read as "x is greater than 4." Since all numbers to the right of 4 are greater than 4, you can draw an arrow from 4 to the right. Since 4 is not greater than itself, use an open circle on 4.



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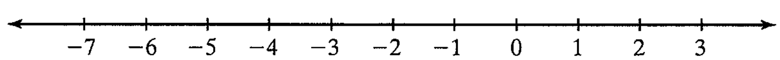
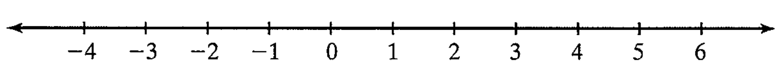
1. **Graph the inequality $x \leq -3$.**
   - a. Write the inequality in words. ______________________
   - b. Will the circle at −3 be open or closed? ______________________
   - c. Graph the solution.



2. **Graph the inequality $x < 3$.**
   - a. Write the inequality in words. ______________________
   - b. Will the circle at 3 be open or closed? ______________________
   - c. Graph the solution.



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Reteaching 4-2

Solving Inequalities by Adding or Subtracting

To solve an inequality you can add the same number to or subtract it from each side of the inequality.

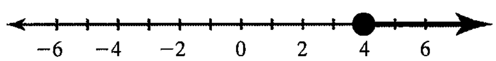
Solve $x + 5 \geq 9$. Graph the solution.

$x + 5 \geq 9$

$x + 5 - 5 \geq 9 - 5$ Subtract 5 from each side.

$x \geq 4$ Simplify.

Graph:



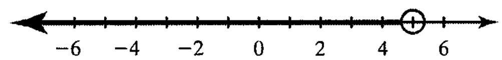
Solve $y - 3 < 2$. Graph the solution.

$y - 3 < 2$

$y - 3 + 3 < 2 + 3$ Add 3 to each side.

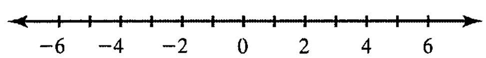
$y < 5$ Simplify.

Graph:

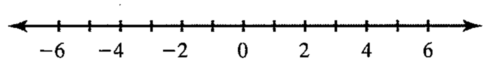


Solve each inequality. Graph the solution.

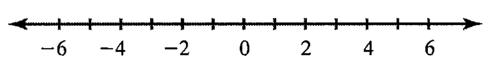
1. $2 + a > 6$ ______________



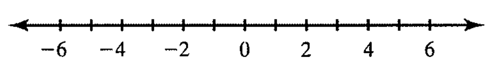
2. $-4 + w \leq 0$ ______________



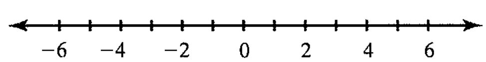
3. $3 + a \geq 8$ ______________



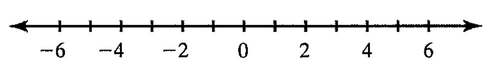
4. $w + 1 \leq 4$ ______________



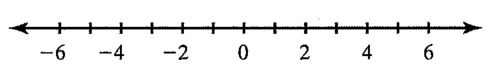
5. $y + 3 < 5$ ______________



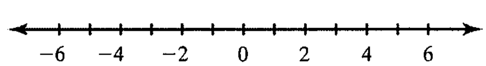
6. $6 + g \geq 12$ ______________



7. $2 + x > 7$ ______________



8. $2 + r < 8$ ______________



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Reteaching 4-3

Solving Inequalities by Multiplying and Dividing

To solve an inequality you can multiply or divide each side by the same number. However, if the number is negative, you must also reverse the direction of the inequality sign.

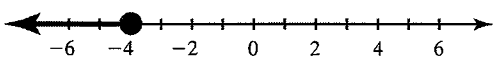
Solve $-4y \geq 16$. Graph the solution.

$-4y \geq 16$

$\frac{-4y}{-4} \leq \frac{16}{-4}$ ← **Divide each side by −4. Reverse the direction of the inequality symbol.**

$y \leq -4$ ← **Simplify.**

Graph:



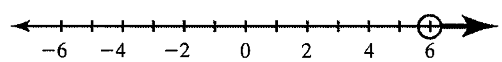
Solve $\frac{w}{3} > 2$. Graph the solution.

$\frac{w}{3} > 2$

$(3)\frac{w}{3} > 2(3)$ ← **Multiply each side by 3.**

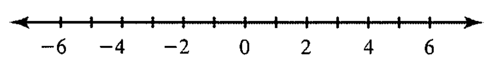
$w > 6$ ← **Simplify.**

Graph:

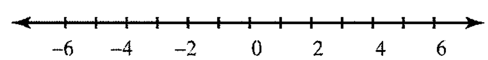


Solve each inequality. Graph the solution.

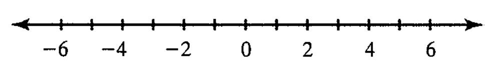
1. $2a > 10$ ______________



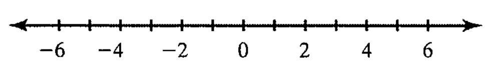
2. $-4w < 16$ ______________



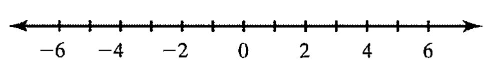
3. $\frac{r}{2} \geq -2$ ______________



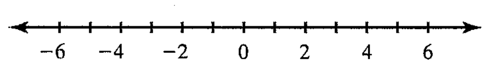
4. $\frac{a}{3} < 1$ ______________



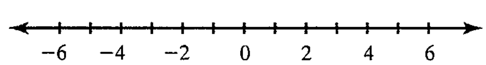
5. $6g < 6$ ______________



6. $-3x \geq -6$ ______________



7. $-\frac{m}{2} > 0$ ______________



Reteaching 4-4

Solving Two-Step Inequalities

You can solve a two-step inequality by using inverse operations and the properties of inequality to get the variable alone on one side of the inequality. For many inequalities, first you undo the addition or subtraction, then you undo the multiplication or division—just like when you solve a two-step equation.

Undoing Addition First

$\frac{x}{2} + 4 \geq -8$

$\frac{x}{2} + 4 - 4 \geq -8 - 4$ ← **Subtract 4 from each side.**

$\frac{x}{2} \geq -12$ ← **Simplify.**

$2 \cdot \frac{x}{2} \geq 2 \cdot (-12)$ ← **Multiply each side by 2.**

$x \geq -24$ ← **Simplify.**

Undoing Subtraction First

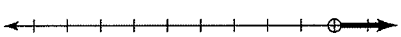
$-2.4y - 7 < -16.6$

$-2.4y - 7 + 7 < -16.6 + 7$ ← **Add 7 to each side.**

$-2.4y < -9.6$ ← **Simplify.**

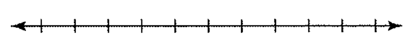
$\frac{-2.4y}{-2.4} > \frac{-9.6}{-2.4}$ ← **Divide each side by −2.4.**

$y > 4$ ← **Simplify.**

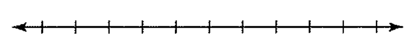


Solve each inequality and graph the solution.

1. $2a + 4 > 12$ ______________________



2. $-3r - 8 > 4$ ______________________



Name ______________________ Class ______________ Date ______________

Reteaching 4-4 (continued)

Solving Two-Step Inequalities

3. $-\frac{1}{3}n + 8 < 9$ ______________

-5 -4 -3 -2 -1 0 1 2 3 4 5

4. $6.25s + 6 \leq 18.5$ ______________

-5 -4 -3 -2 -1 0 1 2 3 4 5

5. $\frac{m}{4} - 3 < -2$ ______________

-5 -4 -3 -2 -1 0 1 2 3 4 5

6. $-5.6q - 3.2 \leq -8.8$ ______________

-5 -4 -3 -2 -1 0 1 2 3 4 5

7. $-\frac{4}{7}x + \frac{3}{7} \leq 1\frac{4}{7}$ ______________

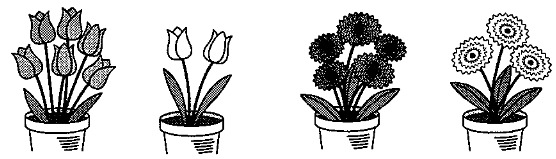
-5 -4 -3 -2 -1 0 1 2 3 4 5

Name ______________________ Class ____________ Date ____________

Reteaching 5-1

Ratios

A ratio is a comparison of two numbers by division. You can write a ratio three ways.



Compare the number of red tulips to the number of yellow tulips.

6 to 2, 6 : 2, or $\frac{6}{2}$

To find equal ratios, multiply or divide each part of the ratio by the same nonzero number.

$\frac{6}{2} = \frac{6 \times 2}{2 \times 2} = \frac{12}{4}$ ← **Multiply by 2.**

The ratio $\frac{3}{1}$ is in **simplest form.**

$\frac{6}{2} = \frac{6 \div 2}{2 \div 2} = \frac{3}{1}$ ← **Divide by 2.**

Use the drawings at the top of the page. Write each ratio in three ways.

1. yellow tulips to red tulips ______________

2. white mums to orange mums ______________

3. red tulips to orange mums ______________

4. yellow tulips to white mums ______________

5. red tulips to all flowers ______________

6. orange mums to all flowers ______________

7. tulips to mums ______________

8. white mums to tulips ______________

9. yellow tulips to all flowers ______________

10. yellow tulips to orange mums ______________

Write two ratios equal to the given ratio.

11. 2 : 5 ______________

12. 18 to 30 ______________

Write each ratio in simplest form.

13. 10 : 15 ______________

14. $\frac{48}{24}$ ______________

15. $\frac{6}{100}$ ______________

16. 8 : 18 ______________

Name ______________________ Class ______________ Date ______________

Reteaching 5-2

Unit Rates and Proportional Reasoning

A **rate** is a ratio that compares two quantities measured in different units.

The cost for 10 copies is $1.50.

The rate is $1.50/10 copies ($1.50 per 10 copies).

A **unit rate** is a rate that has a denominator of 1. You can compare using unit rates.

To find the unit rate for 10 copies:

$$\$1.50/10 \text{ copies} = \frac{\$1.50}{10} = \frac{\$1.50 \div 10}{10 \div 10} = \frac{\$.15}{1}$$

The unit rate is $0.15 per copy. This is also the *unit price.*

COPY CENTER Color Copies	
1 copy	$0.25
10 copies	$1.50
25 copies	$2.50
50 copies	$4.50
100 copies	$6.00

For the better buy, compare unit rates.

The unit price for 10 copies is $0.15/copy.

The unit price for 1 copy is $0.25/copy.

Since $0.15 < $0.25, the 10-copy price is the better buy.

Use the Copy Center chart. Find the unit rate.

1. 25 copies
$\frac{\$2.50}{25} = \frac{\$2.50 \div \square}{25 \div \square} =$ ______________

2. 100 copies
$\frac{\$6.00}{100} = \frac{\$6.00 \div \square}{100 \div \square} =$ ______________

3. 50 copies
$\frac{\$4.50}{50} = \frac{\$4.50 \div \square}{50 \div \square} =$ ______________

Write the unit rate for each situation.

4. drive 1,800 mi in 30 h ______________

5. 390 mi on 15 gal of gasoline ______________

6. jog 4,000 m in 12 min ______________

7. $25.50 for 17 tickets ______________

Find each unit price. Then determine the better buy.

8. juice: 18 oz for $1.26
8 oz for $.70
____

9. cloth: 2 yd for $3.15
6 yd for $7.78
____

10. socks: 2 pairs for $3.50
6 pairs for $9.00
____

11. pecans: 1 lb for $4.80
2 oz for $1.00
____

Name ______________________ Class ______________ Date ______________

Reteaching 5-3

Proportions

A **proportion** is an equation stating that two ratios are equal.

Consider $\frac{2}{10}$ and $\frac{5}{25}$.

$\frac{2}{10} = \frac{2 \div 2}{10 \div 2} = \frac{1}{5}$

$\frac{5}{25} = \frac{5 \div 5}{25 \div 5} = \frac{1}{5}$

Both ratios are equal to $\frac{1}{5}$, the ratios are proportional.

If two ratios form a proportion, the **cross products** are equal.

$\frac{100}{2} = \frac{200}{4}$

$\frac{100}{2} = \frac{200}{4}$

$100 \cdot 4 = 200 \cdot 2$

$400 = 400$

Complete the cross products to determine which pairs of ratios could form a proportion. Then write *yes* or *no*.

1. $\frac{3}{10} \stackrel{?}{=} \frac{6}{20}$

   $3 \cdot 20 =$ ____________

   $10 \cdot 6 =$ ____________

   ____

2. $\frac{12}{24} \stackrel{?}{=} \frac{2}{4}$

   $12 \cdot 4 =$ ____________

   $24 \cdot \square =$ ____________

   ____

3. $\frac{8}{5} \stackrel{?}{=} \frac{16}{8}$

   $8 \cdot \square =$ ____________

   $5 \cdot \square =$ ____________

   ____

Determine if the ratios in each pair are proportional.

4. $\frac{25}{35}, \frac{5}{7}$ ____________

5. $\frac{15}{3}, \frac{10}{2}$ ____________

6. $\frac{9}{3}, \frac{12}{4}$ ____________

7. $\frac{2}{5}, \frac{6}{15}$ ____________

8. $\frac{3.6}{200}, \frac{1.8}{100}$ ____________

9. $\frac{6}{12}, \frac{4}{8}$ ____________

10. $\frac{16}{11}, \frac{96}{24}$ ____________

11. $\frac{3}{7}, \frac{2}{5}$ ____________

12. $\frac{2}{22}, \frac{1}{11}$ ____________

Name ______________________ Class ______________ Date ______________

Reteaching 5-4

Solving Proportions

Solving a proportion means finding a missing part of the proportion. You can use unit rates to solve a proportion. First find the unit rate. Then multiply to solve the proportion.

Shawn filled 8 bags of leaves in 2 hours. At this rate, how many bags would he fill in 6 hours?

① Find a unit rate for the number of bags per hour. Divide by the denominator.

$\frac{8 \text{ bags}}{2 \text{ hours}} = \frac{8 \text{ bags} \boxed{\div 2}}{2 \text{ hours} \boxed{\div 2}} = \frac{4 \text{ bags}}{1 \text{ hour}}$ The unit rate is 4 bags per hour.

② Multiply the unit rate by 6 to find the number of bags he will fill in 6 hours.

Unit rate		Number of hours		Total
↓		↓		↓
4	×	6	=	24

At this rate, Shawn can fill 24 bags in 6 hours.

If two ratios form a proportion, the **cross products** are equal.

Solve. $\frac{5}{15} = \frac{n}{3}$

① Write the cross products. $5 \cdot 3 = 15 \cdot n$

② Simplify. $15 = 15n$

③ Solve the equation. $n = 1$

Solve.

1. The bookstore advertises 5 notebooks for $7.75. At this rate, how much will 7 notebooks cost?

____

2. Leroy can lay 144 bricks in 3 hours. At this rate, how many bricks can he lay in 7 hours?

____

Solve each proportion using cross products.

3. $\frac{4}{24} = \frac{n}{6}$

4 · ______ = 24 · ______

n = ______

4. $\frac{30}{5} = \frac{6}{n}$

______ = ______

n = ______

5. $\frac{n}{6} = \frac{27}{9}$

______ = ______

n = ______

Solve each proportion.

6. $\frac{4}{10} = \frac{n}{15}$ ______________

7. $\frac{4}{200} = \frac{n}{100}$ ______________

8. $\frac{6}{n} = \frac{5}{10}$ ______________

9. $\frac{32}{22} = \frac{96}{n}$ ______________

10. $\frac{6}{3} = \frac{n}{5}$ ______________

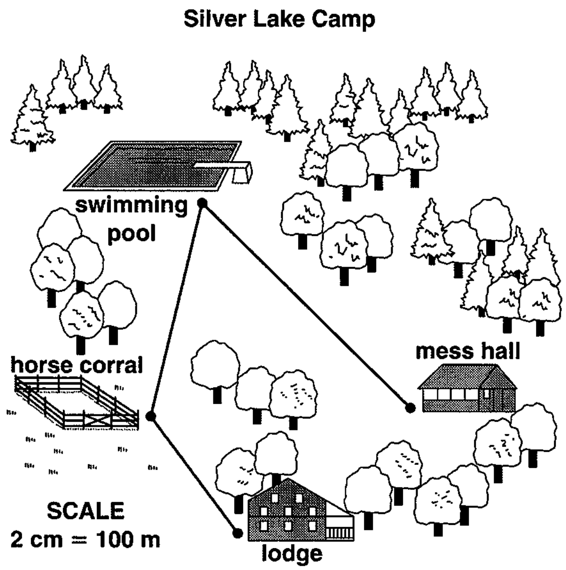
11. $\frac{2}{n} = \frac{4}{10}$ ______________

Name ______________________ Class ______________ Date ______________

Reteaching 5-6

Maps and Scale Drawings

A **scale drawing** is an enlarged or reduced drawing of an object. A map is a scale drawing. On this map, the pool is 3 cm from the horse corral. What is the actual distance from the corral to the pool?



① Use the scale. Write a ratio of distance on the map to actual distance.

$$\frac{\text{map (cm)}}{\text{actual (m)}} = \frac{2}{100}$$

② Write a proportion using the scale.

$$\frac{\text{map (cm)}}{\text{actual (m)}} = \frac{2}{100} = \frac{3}{n}$$

③ Use cross products. Solve for n.

$$2n = 100 \cdot 3$$
$$n = 150 \text{ m}$$

The pool is 150 m from the corral.

Use the information on the map. Write and solve a proportion to find the distance.

1. On the map, the mess hall is 4 cm from the pool. What is the actual distance from the pool to the mess hall?

$$\frac{\text{map}}{\text{actual}} = \frac{\square}{100} = \frac{\square}{n}$$

$n =$ ________

2. The lodge is 2 cm from the horse corral on the map. What is the actual distance from the corral to the lodge?

$$\frac{\text{map}}{\text{actual}} = \frac{\square}{100} = \frac{\square}{n}$$

$n =$ ________

3. The pool is actually 225 m from the lodge. How far would the pool be from the lodge on the map?

$$\frac{\text{map}}{\text{actual}} = \frac{\square}{100} = \frac{n}{\square}$$

$n =$ ________

4. The mess hall is 150 m from the lodge. How far would the mess hall be from the lodge on the map?

$$\frac{\text{map}}{\text{actual}} = \frac{\square}{100} = \frac{\square}{n}$$

$n =$ ________

5. A volleyball court will be built 175 m from the lodge. How far would the volleyball court be from the lodge on the map?

____

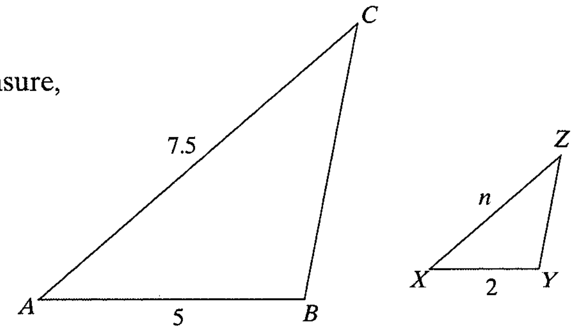
Name Class ________________ Date ________________

Reteaching 5-5

Similar Figures

Two polygons are **similar** if

- corresponding angles have the same measure, and
- the lengths of corresponding sides are proportional.



$\triangle ABC \sim \triangle XYZ$

You can use proportions to find missing lengths in similar (~) figures.

① Find corresponding sides.

$\overline{AB}$ corresponds to $\overline{XY}$.
$\overline{AC}$ corresponds to $\overline{XZ}$.
$\overline{BC}$ corresponds to $\overline{YZ}$.

② Write ratios of their lengths in a proportion. $\frac{AB}{XY} = \frac{AC}{XZ}$

③ Substitute the information you know. $\frac{5}{2} = \frac{7.5}{n}$

④ Write cross products. Solve for *n*.

$5n = 2 \cdot 7.5$

$n = 3$

The length of $\overline{XZ}$ is 3 units.

The figures are similar. Find the corresponding sides. Then complete the proportion and solve for *n*.

1. $\overline{AB}$ corresponds to ________.

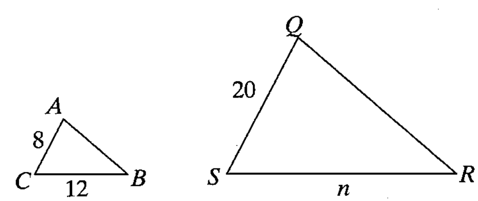
   $\overline{BC}$ corresponds to ________.

   $\overline{CA}$ corresponds to ________.

2. $\frac{CA}{SQ} = \frac{\square}{RS}$

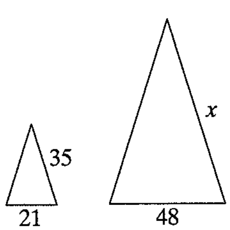
   $\frac{8}{20} = \frac{\square}{\square}$

   $n =$ ________

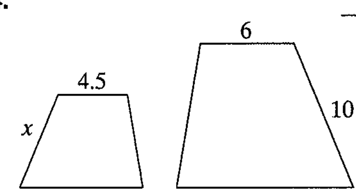


The pairs of figures below are similar. Find the value of each variable.

3. ________



4. ________



Name ______________________ Class ______________ Date ______________

Reteaching 5-7

Proportional Relationships

You can use a table to determine if there is a proportional relationship.

Compare the ratios to see if there is a proportional relationship.

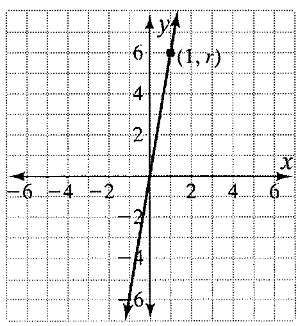
Hours	2	4	5
Pages	12	24	30

$\frac{12}{2} = \frac{24}{4} = \frac{30}{5}$ ← There is a proportional relationship between pages and hours.

Hours	2	3	4
Pages	10	14	24

$\frac{10}{2} \neq \frac{14}{3} \neq \frac{24}{4}$ ← There is not a proportional relationship between pages and hours.

You can use a graph to find the unit rate.



The line passes through (0, 0) and (2, 12), therefore it passes through (1, 6).

Since $r = 6$, then the unit rate is 6 pages/hour.

You can use a ratio to find the unit rate.

The value of the ratio of quantities in a proportional relationship is called the **constant of proportionality**, which is equivalent to the unit rate.

$\frac{\text{pages}}{\text{hour}} = \frac{12}{2}$ ← **Find the pages per hour by dividing the number of pages by the number of hours.**

$= 6$ ← **Simplify.**

The constant of proportionality is 6. The unit rate is 6 pages/hour.

Determine whether each table represents a proportional relationship. If so, find the constant of proportionality.

1.

x	4	5	7
y	\$10.40	\$13.00	\$17.50

____

2.

x	3	4	5
y	$\frac{21}{4}$	7	$\frac{35}{4}$

____

Name ______________________ Class ______________ Date ______________

Reteaching 6-1

Percents, Fractions, and Decimals

To write a percent as a fraction, write a fraction with 100 as the denominator.

$45\% = \frac{45}{100}$ ← **Denominator 100**

$= \frac{45 \div 5}{100 \div 5} = \frac{9}{20}$ ← **Simplify.**

$45\% = \frac{9}{20}$

To write a decimal as a percent, multiply by 100.	To write a percent as a decimal, divide by 100.
Write 0.85 as a percent.	Write 46% as a decimal.
$0.85 \cdot 100 = 85$	$46 \div 100 = 0.46$
$0.85 = 85\%$	$46\% = 0.46$

Write each fraction as a percent.

1. $\frac{3}{4}$ ______
2. $\frac{12}{25}$ ______
3. $\frac{4}{5}$ ______
4. $\frac{23}{4}$ ______

Write each percent as a fraction in simplest form.

5. 45% ______
6. 60% ______
7. 16% ______
8. 25% ______
9. 37.5% ______
10. 99% ______
11. 40% ______
12. 86% ______

Write each percent as a decimal or each decimal as a percent.

13. 35% ______
14. 48% ______
15. 8% ______
16. 12% ______
17. 5.5% ______
18. 0.6% ______
19. 0.39 ______
20. 0.735 ______
21. 0.34 ______
22. 0.4 ______
23. 0.6 ______
24. 6 ______

Name ______________________ Class ______________ Date ______________

Reteaching 6-2

Solving Percent Problems Using Proportions

You can use proportions to solve percent problems. Remember, the percent is compared to 100.

Finding the part:	Finding the whole:	Finding the percent:
10% of 40 is _?_.	20% of _?_ is 8.	_?_ % of 25 is 20.
$\frac{10}{100} = \frac{n}{40}$	$\frac{20}{100} = \frac{8}{n}$	$\frac{n}{100} = \frac{20}{25}$
$100 \cdot n = 10 \cdot 40$	$20 \cdot n = 100 \cdot 8$	$25 \cdot n = 100 \cdot 20$
$n = 4$	$n = 40$	$n = 80$
10% of 40 is 4.	20% of 40 is 8.	80% of 25 is 20.

Complete to solve for *n*.

1. 75% of _?_ is 12.

$\frac{75}{100} = \frac{12}{n}$

$75 \cdot$ ______ $= 100 \cdot$ ______

$n =$ ______

2. 20% of _?_ is 82.

$\frac{20}{100} = \frac{82}{\square}$

$20 \cdot$ ______ $= 100 \cdot$ ______

$n =$ ______

3. 5% of _?_ is 9.

$\frac{5}{100} = \frac{\square}{n}$

______ = ______

$n =$ ______

4. 60 is 5% of n.

$\frac{5}{100} = \frac{\square}{n}$

$5n = 100 \cdot$ ______

$n =$ ______

5. 6% of n is 4.8.

$\frac{6}{\square} = \frac{\square}{n}$

$6n =$ ______ $\cdot 4.8$

$n =$ ______

6. 51 is 170% of n.

$\frac{\square}{100} = \frac{\square}{n}$

______ = ______

$n =$ ______

Use a proportion to solve.

7. 12% of n is 9. ______________

8. 49% of n is 26.95. ______________

9. 18% of n is 27. ______________

10. What is 210% of 44? ______________

11. What is 30% of 200? ______________

12. 64 is what percent of 80? ______________

Name ______________________ Class ______________ Date ______________

Reteaching 6-3

Solving Percent Problems Using Equations

You can write equations to solve percent problems by substituting amounts into the statement: "______% of ______ is ______?"

- 64% of 50 is what number?

①	Choose a variable for the unknown amount.	Let n = unknown number.
②	Reword the statement, ______% of ______ is ______.	64% of 50 is n
③	Write an equation.	$64\% \cdot 50 = n$
④	Write the percent as a decimal.	$0.64 \cdot 50 = n$
⑤	Multiply to solve for n.	$32 = n$
⑥	So, 64% of 50 is 32.	

- What percent of 36 is 18?

①	Choose a variable for the unknown amount.	Let p = unknown percent.
②	Reword the statement, ______% of ______ is ______.	p% of 36 is 18.
③	Write an equation.	$36 \cdot p = 18$
④	Divide each side by 36.	$36 \cdot \frac{p}{36} = \frac{18}{36}$
⑤	Simplify and write the decimal as a percent.	$p = 0.5 = 50\%$
⑥	So, 18 is 50% of 36.	

Answer each question.

1. Write an equation for: 9% of 150 is what number. ______ · ______ = n
2. Solve the equation to find 9% of 150 is what number? __________
3. 48% of 250 is what number? __________
4. 82% of 75 is what number? __________
5. 32% of 800 is what number? __________
6. Reword the statement: What percent of 75 is 12? ______ % of ______ is ______
7. Use the statement to find what percent of 75 is 12. __________
8. What percent of 60 is 18? __________
9. What percent of 50 is 35? __________

Name ______________________ Class ______________ Date ______________

Reteaching 6-4

Applications of Percent

Finding Sales Tax

sales tax = percent of tax · purchase price

Find the amount of sales tax on a television that costs $350 with an 8% sales tax.

sales tax = 8% · $350
sales tax = 0.08 · 350
sales tax = 28

The sales tax is $28.

How much does the television cost with sales tax?

$350 + $28 = $378

Finding a Commission

commission = commission rate · sales

Find the commission earned with a 3% commission rate on $3,000 in sales.

commission = 3% · $3,000
commission = 0.03 · 3,000
commission = 90

The commission earned is $90.

How much do you earn if you have a base salary of $500 plus 3% commission on sales of $3,000?

$90 + $500 = $590

Find the total cost.

1. $10.00 with a 4% sales tax ______________

2. $8.75 with a 5.25% sales tax ______________

3. $61.00 with a 7% sales tax ______________

4. $320.00 with a 6.5% sales tax ______________

5. $6.30 with an 8% sales tax ______________

6. $26.75 with a 7.5% sales tax ______________

Find each commission.

7. 6% on $3,000 in sales ______________

8. 1.5% on $400,000 in sales ______________

9. 8% on $1,200 in sales ______________

10. 5.5% on $2,400 in sales ______________

Name ______________________ Class ______________ Date ______________

Reteaching 6-5

Simple Interest

When you deposit money in a bank, the bank pays interest. **Simple interest** is interest paid only on the amount you deposited, called the **principal.**

Simple Interest

To find simple interest, use this formula.

Interest = principal · rate · time in years

$I = p \cdot r \cdot t$

Find the simple interest on $1,800 invested at 5% annual interest for 3 years.

$I = p \cdot r \cdot t$
$= 1{,}800 \cdot 0.05 \cdot 3$ ← **Use 0.05 for 5%.**
$= 270$

The interest is $270. (The balance will be $1,800 + $270, or $2,070.)

Find the simple interest earned by each account.

1. $800 principal
   4% interest rate
   5 years
   $I = p \cdot r \cdot t$
   = ________ · ________ · ________
   = ________

2. $800 principal
   3% interest
   4 years
   ____

3. $1,900 principal
   4.5% interest
   20 years
   ____

4. $20,000 principal
   3.5% interest
   15 years
   ____

Name ______________________ Class ______________ Date ______________

Reteaching 6-6

Finding Percent of Change

Percent of change is the percent something increases or decreases from its original amount.

	Find the percent of increase from 12 to 18.	Find the percent of decrease from 20 to 12.
① Subtract to find the amount of change.	$18 - 12 = 6$	$20 - 12 = 8$
② Write a proportion. $\frac{\text{change}}{\text{original}} = \frac{\text{percent}}{100}$	$\frac{6}{12} = \frac{n}{100}$	$\frac{8}{20} = \frac{n}{100}$
	$6 \cdot 100 = 12n$	$8 \cdot 100 = 20n$
③ Solve for n.	$n = 50$	$n = 40$
	The percent of increase is 50%.	The percent of decrease is 40%.

State whether the change is an *increase* or *decrease.* Complete to find the percent of change.

1. 40 to 60

$60 - 40 =$ ______

$\frac{\square}{40} = \frac{n}{100}$

______ $\cdot 100 = 40n$

$n =$ ______

______

2. 15 to 9

$15 - 9 =$ ______

$\frac{\square}{15} = \frac{n}{100}$

______ $\cdot 100 = 15n$

$n =$ ______

______

3. 0.4 to 0.9

$0.9 - 0.4 =$ ______

$\frac{\square}{0.4} = \frac{n}{\square}$

______ $= 0.4n$

$n =$ ______

______

Find the percent of *increase.*

4. 16 to 40 ______

5. 20 to 22 ______

6. 9 to 18 ______

7. 28 to 35 ______

8. 80 to 112 ______

9. 150 to 165 ______

Find the percent of *decrease.*

10. 20 to 15 ______

11. 100 to 57 ______

12. 52 to 26 ______

13. 140 to 126 ______

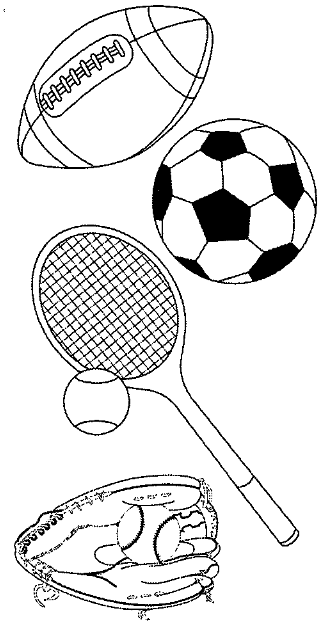
14. 75 to 72 ______

15. 1,000 to 990 ______

Name ______________________ Class ______________ Date ______________

Reteaching 8-1

Random Samples and Surveys



Carlos is curious about sports that students in his school like best. He cannot interview every student in the school. But he could interview a sample of the school **population.**

Carlos wants a **random sample.** A sample is random if everyone has an equal chance of being selected. How will Carlos get a random sample? He considers two possibilities:

- He can interview 30 students at a soccer game.
- He can interview 5 students in each of 6 class changes.

Carlos realizes that students at a soccer game probably like soccer better than other sports. That would not be a random sample. He decides on the interviews during class changes.

What question will he ask? He considers two possibilities:

- "Which sport do you prefer, football, soccer, baseball, or tennis?"
- "Which do you enjoy most, the slow sport of baseball or one of the more exciting sports like football, soccer, or tennis?"

The second question is **biased.** It makes one answer seem better than another. Carlos decides to ask the first question.

---

1. You want to find how many people in your community are vegetarian. Where would be the best place to take a survey?

____

Is each question biased or fair?

2. Will you vote for the young, inexperienced candidate, Mr. Soong, or the experienced candidate, Ms. Lopez? ______________

3. Will you vote for Mr. Soong or Ms. Lopez? ______________

You plan to survey people to see what percent own their home and what percent rent. Tell whether the following will give a random sample. Explain.

4. You interview people outside a pool supply store in the suburbs.

____

5. You interview people in the street near an apartment complex.

____

6. You mail a survey to every 20th person in the telephone book.

____

Reteaching 8-2

Estimating Population Size

Researchers tagged 100 fish in a pond and then released them back into the pond. Later they captured 60 fish and found that 3 were tagged. Estimate the number of fish in the pond.

① Write a proportion of tagged fish to total fish. $\frac{\text{tagged fish (pond)}}{\text{total fish (pond)}} = \frac{\text{tagged fish (sample)}}{\text{total fish (sample)}}$

$\frac{100}{n} = \frac{3}{60}$

② Write cross products. $3n = 6{,}000$

③ Solve. $n = 2{,}000$

There are about 2,000 fish in the pond.

Complete to estimate the number of deer in the woods.

1. One year researchers tagged 80 deer. They later captured 15 deer and found 5 were tagged. Estimate the number of deer in the woods.

   $\frac{80}{n} = \frac{\square}{\square}$

   $5n = 15 \cdot$ ________

   $n =$ ________

   There are about ________ deer.

2. Two years later, researchers tagged 45 deer. They later captured 20 and found 3 were tagged. Estimate the number of deer in the woods then.

   $\frac{\square}{n} = \frac{\square}{20}$

   $3n =$ ________

   $n =$ ________

   There are about ________ deer.

Use a proportion to estimate each animal population.

3. In another project, researchers caught and tagged 85 sea lions in a bay. Later they caught and released 50 sea lions. Of those, 9 had tags. Estimate the sea lion population in the bay.

   ____

4. Other researchers caught and tagged 5 spotted owls. Later they caught 7 owls. Of those, 4 were tagged. Estimate the number of spotted owls in that forest.

   ____

5. An ecology class helped researchers determine the rabbit population in a nature preserve. One weekend, the students captured, tagged, and set free 32 rabbits. A month later, they captured 27 rabbits, including 16 with tags. Estimate the number of rabbits in the nature preserve.

   ____

Reteaching 8-3

Inferences

Example 1.

To find the average, or mean, of a set of data, you divide the sum of the data values by the number of data values.

You can use information about a sample population to draw inferences about the whole population.

A teacher selected a random sample of 25 middle school students and surveyed them about the number of hours a night they spend on homework. Based on the sample, what is the best estimate of the mean number of hours that middle school students in that school spend on homework every night?

Random Sample of Number of Hours Spent on Homework per Student			
2	4	2	5
4	4	5	5
4	1	1	3
3	3	3	3
2	3	1	2

Step 1 Find the average of the sample data.

$$\text{mean} = \frac{\text{sum of the data values}}{\text{number of data values}}$$

$$= \frac{60}{20}$$

$$= 3$$

Step 2 Use the average of the sample data to draw an inference.

The average time spent on homework in the sample is 3 hours, so the average time spent on homework by all middle school students is likely close to 3.

Example 2.

You can compare random samples to see how much estimates or predictions you make from the samples will vary.

There are 410 students in Jefferson Middle School. Anna, Bryan, and Chris each survey a random sample of 25 students about which elective they would like to see added to the curriculum. Their results are shown in the table.

Votes for Electives			
Elective	**Anna's Sample**	**Bryan's Sample**	**Chris's Sample**
Computer Science	12	7	5
Woodworking	9	8	10
Art History	4	10	11

Reteaching 8-3 (continued)

Inferences

For each sample, predict how many students in the school would choose computer science.

Step 1 Use a proportion to make a prediction for each sample:

$$\frac{\text{votes for computer science in sample}}{\text{students in sample}} = \frac{\text{predicted votes for computer science}}{\text{students in school}}$$

Anna's Sample:

$\frac{12}{25} = \frac{x}{410}$

$12 \times 410 = 25x$

$4{,}920 = 25x$

$197 \approx x$

about 197 votes

Bryan's Sample:

$\frac{7}{25} = \frac{x}{410}$

$7 \times 410 = 25x$

$2{,}870 = 25x$

$115 \approx x$

about 115 votes

Chris's Sample:

$\frac{5}{25} = \frac{x}{410}$

$5 \times 410 = 25x$

$2{,}050 = 25x$

$82 \approx x$

about 82 votes

Step 2 Describe the variation in the predictions.

The greatest prediction is 197 votes, and the least prediction is 82 votes. So the predictions vary by $197 - 82 = 115$ votes.

Step 3 Draw an inference about the number of votes computer science will get.

Computer science is likely to get about 115 votes, the median prediction.

The table shows the ages of a random sample of 25 customers at a food truck. Use the sample to draw an inference about each measure. Support your answer.

Random Sample of Ages				
14	24	22	38	27
23	19	42	24	39
46	10	11	42	37
34	41	5	19	5
43	50	22	20	43

1. The mean age of customers at the truck ______________
2. The percent of customers under age 21 ______________
3. The percent of customers age 40 and over ______________
4. The median age of customers at the food truck ______________

Name ______________________ Class ______________ Date ______________

Reteaching 8-3 (continued)

Inferences

Use the preceding table to answer questions 5 and 6.

5. a. For each sample, predict how many students in the school will choose woodworking as an elective.

   ____

   ____

   b. Describe the variation in the predictions.

   ____

   ____

   c. Draw an inference about the number of students in the school who will choose woodworking as an elective.

   ____

   ____

6. a. For each sample, predict how many students in the school will choose art history as an elective.

   ____

   ____

   b. Describe the variation in the predictions.

   ____

   ____

   c. Draw an inference about the number of students in the school who will choose art history as an elective.

   ____

   ____

Name ______________________ Class ______________ Date ______________

Reteaching 8-4

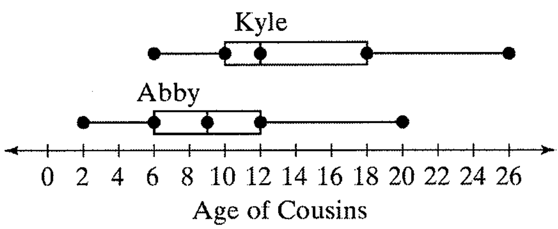
Data Variability

You can use random samples to compare two populations.

With a box plot, you can visually see the overlap of two data sets.

The interquartile range (IQR) is one measure of variability, or how much a data set is spread out.

This box plot shows the ages of Abby's and Kyle's cousins. Compare the IQRs of the data sets to draw an inference about the ages of the cousins.



IQR for Kyle:
$18 - 10 = 8$

IQR for Abby:
$12 - 6 = 6$

The IQR for Kyle's data set is larger than Abby's. So you can infer that the ages of Kyle's cousins vary more than the ages of Abby's cousins.

You can also use the mean absolute deviation (MAD), which is the average distance between the mean and each data value.

Expressing the difference between their centers as a multiple of the MAD helps you determine the amount of overlap between the two sets. A multiple less than 1 indicates a large amount of overlap. A multiple greater than 1 indicates there is little or no amount of overlap.

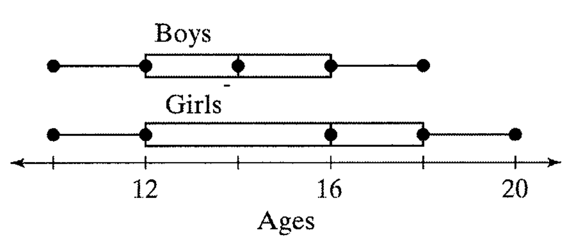
Step 1: Calculate the means of each data set.

Step 2: Calculate the MAD of each data set.

Step 3: Find the multiple n of the MAD that equals the difference between the means: MAD $\cdot\, n$ = difference of means.

Step 4: Make an inference.

The box plot shows the ages of a random sample of boys and girls at the mall. Use the data for Exercises 1–3.



1. Determine the IQR of each data set.

 ____

2. Determine the medians of the data sets.

 ____

3. Determine the greatest age of boys.

 ____

Name ________________ Class ________________ Date ________________

Reteaching 8-4 (continued)

Data Variability

The lists below show the ages of a random sample of children at story hour at the library in two towns. Use the data for Exercises 4–7.

Avon

4, 5, 2, 1, 2, 5, 4, 4, 3, 3

Farmingdale

3, 3, 5, 4, 5, 3, 5, 5, 2, 2

4. Compare the means of the data sets, and use the comparison to draw an inference.

____

5. Determine the MAD for each data set.

____

6. What multiple *n* of the MAD equals the difference between the means?

____

7. What does this tell you about the overlap of the data sets?

____

Name ________________ Class ________________ Date ________________

Reteaching 9-1

Probability

To find a **theoretical probability,** first list all possible **outcomes.** Then use the formula:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

A letter is selected at random from the letters of the word FLORIDA. What is the probability that the letter is an A?

- There are 7 letters (possible outcomes).
- There is one A, which represents a favorable outcome.

$$P(\text{A}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}} = \frac{1}{7}$$

The probability that the letter is an A is $\frac{1}{7}$.

Selecting a letter other than A is called *not* A and is the **complement** of the event A. The sum of the probabilities of an event and its complement equals 1, or 100%.

What is the probability of the event "*not* A"?

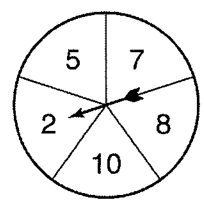
$$P(\text{A}) + P(\textit{not}\ \text{A}) = 1$$

$$\frac{1}{7} + P(\textit{not}\ \text{A}) = 1$$

$$P(\textit{not}\ \text{A}) = 1 - \frac{1}{7} = \frac{6}{7}$$

The probability of the event "*not* A" (selecting F, L, O, R, I, or D) is $\frac{6}{7}$.

Spin the spinner shown once. Find each probability as a fraction, a decimal, and a percent.



1. $P(5)$

$\frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$

$= \frac{\square}{5}$ ________________

2. $P(\text{odd number})$

$\frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$

$= \frac{2}{\square}$ ________________

You select a card at random from a box that contains cards numbered from 1 to 10. Find each probability as a fraction, a decimal, and a percent.

3. $P(\text{even number})$ ________________

4. $P(\text{number less than } 4)$ ________________

5. $P(\text{not } 5)$ ________________

The letters H, A, P, P, I, N, E, S, and S are written on pieces of paper. Select one piece of paper. Find each probability.

6. $P(\text{not vowel})$ ________________

7. $P(\text{not E})$ ________________

A number is selected at random from the numbers 1 to 50. Find the odds in favor of each outcome.

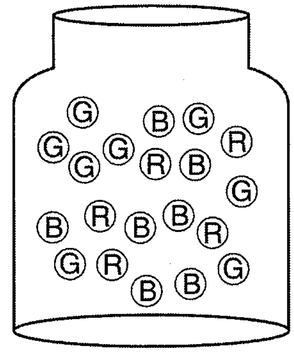
8. selecting a multiple of 5 ________________

9. selecting a factor of 50 ________________

Name ______________________ Class ______________ Date ______________

Reteaching 9-2

Experimental Probability



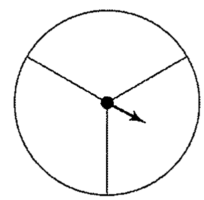
Probability measures how likely it is that an event will occur. For an **experimental probability,** you collect data through observations or experiments and use the data to state the probability.

The jar contains red, green, and blue chips. You shake the jar, draw a chip, note its color, and then put it back. You do this 20 times with these results: 7 blue chips, 5 red chips, and 8 green chips. The experimental probability of drawing a green chip is

$$P(\text{green chip}) = \frac{\text{number of times "green chips" occur}}{\text{total number of trials}}$$

$$P(\text{green chip}) = \frac{8}{20} = \frac{2}{5} = 0.4 = 40\%$$

The probability of drawing a green chip is $\frac{2}{5}$, or 0.4, or 40%.



Sometimes a model, or simulation, is used to represent a situation. Then, the simulaton is used to find the experimental probability. For example, spinning this spinner can simulate the probability that 1 of 3 people is chosen for president of the student body.

Use the 20 draws above to complete each exercise.

1. What is the experimental probability of drawing a red chip? Write the probability as a fraction.

   $P(\text{red chip}) = \frac{\square}{20} =$ ______________

2. What is the experimental probability of drawing a blue chip? Write the probability as a percent.

   $P(\text{blue chip}) = \frac{\square}{\square} =$ ______________

Suppose you have a bag with 30 chips: 12 red, 8 white, and 10 blue. You shake the jar, draw a chip, note its color, and then put it back. You do this 30 times with these results: 10 blue chips, 12 red chips, and 8 white chips. Write each probability as fraction in simplest form.

3. $P(\text{red})$ ______________
4. $P(\text{white})$ ______________
5. $P(\text{blue})$ ______________

Describe a probability simulation for each situation.

6. You guess the answers on a true/false test with 20 questions.

   ____

   ____

7. One student out of 6 is randomly chosen to be the homeroom representative.

   ____

   ____

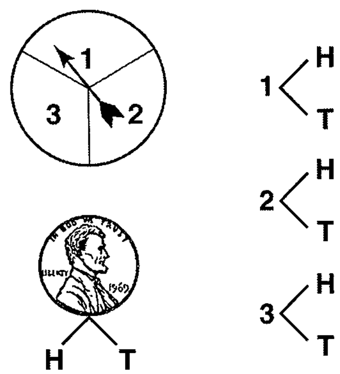
Name ______________________ Class ______________ Date ______________

Reteaching 9-3

Sample Spaces

The set of all possible outcomes of an experiment is called the **sample space.**

You can use a *tree diagram* or a table to show the sample space for an experiment. The tree diagram below shows the sample space for spinning the spinner and tossing a coin.



There are 6 possible outcomes: 1H, 1T, 2H, 2T, 3H, 3T. What is the probability of spinning a 3 and tossing heads? There is one favorable outcome (3H) out of 6 possible outcomes. The probabilty is $\frac{1}{6}$.

You can use the *counting principle* to find the number of possible outcomes: If there are *m* ways of making one choice and *n* ways of making a second choice, then there are $m \times n$ ways of making the first choice followed by the second.

Evelyn and Kara are planning to go skating or to a movie. Afterward they want to go out for pizza, tacos, or cheeseburgers. How many possible choices do they have?

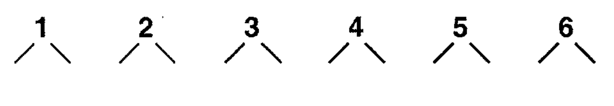
- There are *two choices* for an activity and *three choices* for food.
- First choices × Second choices

$$2 \quad \times \quad 3 \quad = 6$$

There are 6 possible choices.

Complete the tree diagram to show the sample space.

1. Roll a number cube and toss a coin. What is the probability of getting (4, Heads)?



Number of outcomes ______________

$P(4, \text{heads}) =$ ______________

Use the counting principle to find the number of possible outcomes.

2. 4 kinds of yogurt and 8 toppings

____

3. 6 shirts and 9 pairs of slacks

____

4. 3 types of sandwiches and 3 flavors of juice

____

5. 4 types of bread and 6 different sandwich spreads

____

Name ______________________ Class ______________ Date ______________

Reteaching 9-4

Compound Events

If you toss a coin and roll a number cube, the events are **independent.** The outcome of one event does not affect the outcome of the second event.

Find the probability of tossing a heads (H) and rolling an even number (E).

Find P(H and E). H and E are independent.

① Find P(H):

$P(\text{H}) = \frac{1 \text{ heads}}{2 \text{ sides}} = \frac{1}{2}$

② Find P(E):

$P(\text{E}) = \frac{3 \text{ evens}}{6 \text{ faces}} = \frac{1}{2}$

③ $P(\text{H and E}) = P(\text{H}) \times P(\text{E}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

If the outcome of the first event affects the outcome of the second event, the events are **dependent.**

A bag contains 3 blue and 3 red marbles. Draw a marble, then draw a second marble without replacing the first marble. Find the probability of drawing 2 blue marbles.

① Find P(blue).

$P(\text{blue}) = \frac{3 \text{ blue}}{6 \text{ marbles}} = \frac{1}{2}$

② Find P(blue after blue).

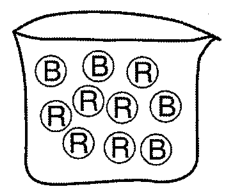
$P(\text{blue after blue}) = \frac{2 \text{ blue}}{5 \text{ marbles}} = \frac{2}{5}$

③ Find P(blue, then blue)

P(blue, then blue)
$= P(\text{blue}) \times P(\text{blue after blue})$

$= \frac{1}{2} \times \frac{2}{5} = \frac{1}{5}$

In Exercises 1–3, you draw a marble at random from the bag of marbles shown. Then, you replace it and draw again. Find each probability.



1. P(blue, then red) ________
2. P(2 reds) ________
3. P(2 blues) ________

Next, you draw two marbles randomly *without* replacing the first marble. Find each probability.

4. P(blue, then red) ________
5. P(2 reds) ________
6. P(2 blues) ________

You draw two letters randomly from a box containing the letters M, I, S, S, O, U, R, and I.

7. Suppose you do not replace the first letter before drawing the second. What is P(M, then I)?

____

8. Suppose you replace the first letter before drawing the second. What is P(M, then I)?

____

Name ______________________ Class ______________ Date ______________

Reteaching 9-5

Simulating Compound Events

Step 1 **Choose a simulation tool.**

- You can use a number cube when there are 6 equally likely outcomes.
- You can use a spinner with x number of equal spaces when there are x equally likely outcomes.
- You can use a coin when there are 2 equally likely outcomes.
- You can use random digits.

Step 2 **Decide which outcomes are favorable.**

- Choose what you need to land on, roll, or toss to get a favorable outcome.

Step 3 **Describe a trial.**

- For each trial, use your simulation tool until you get a favorable outcome.
- Record the number of times you use your tool to get your favorable outcome.

Step 4 **Perform 20 trials. Then estimate the probability.**

- Make a table showing your 20 trials and their outcomes.
- Use that to find the probability: the number of favorable outcomes over 20.

One-fourth of the students in the seventh grade have no siblings. Design a simulation for estimating the probability that you would need to ask at least two students before finding one with no siblings.

1. Choose a simulation tool.

____

____

____

2. Decide which outcomes are favorable.

____

____

____

____

Reteaching 9-5 (continued)

Simulating Compound Events

3. Describe a trial.

____

____

____

4. Perform 20 trials.

Students Asked to Find One with No Siblings	Frequency
1	
2	

5. Find the probability that you would need to ask at least two students before finding one with no siblings.

____